

7.1 Vectors in 2 Dimensions

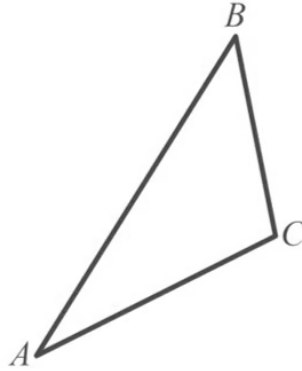
Question Paper

Course	CIEA Level Maths
Section	7. Vectors
Topic	7.1 Vectors in 2 Dimensions
Difficulty	Hard

Time allowed: 60
Score: /48
Percentage: /100

Question 1a

In triangle ABC , $\overrightarrow{AB} = 5\mathbf{i} + 8\mathbf{j}$ and $\overrightarrow{BC} = \mathbf{i} - 5\mathbf{j}$



(a) Explain why $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \mathbf{0}$.

[1 mark]

Question 1b

(b) Find \overrightarrow{CA} and calculate its magnitude.

[3 marks]

Question 2a

(a) $\mathbf{a} = \begin{pmatrix} -1 \\ n \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} m \\ 6 \end{pmatrix}$

Given that the resultant of \mathbf{a} , \mathbf{b} and \mathbf{c} is the zero vector, find the values of m and n .

[2 marks]

Question 2b

(b) $\mathbf{d} = \begin{pmatrix} -3k \\ k \end{pmatrix}$

Given that $|\mathbf{d}| = 2\sqrt{15}$, find two possible values for k . Give your answer as an exact value.

[2 marks]

Question 3

The point A lies on the curve with equation $y = x^2 - 2$. The position vector of A is $\overrightarrow{OA} = 3k\mathbf{i} - 17k\mathbf{j}$, where k is a positive constant. Find the value of k , and hence determine the coordinates of A .

[4 marks]

Question 4

The vectors **a**, **b** and **c** are given as

$$\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -3k \\ k \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

Given that $\mathbf{a} - \mathbf{b}$ is parallel to $\mathbf{a} + \mathbf{c}$ find the value of k .

[4 marks]

Question 5a

Vector \overrightarrow{AB} has a magnitude of $6\sqrt{3}$ and makes an angle of 150° with the positive x -axis.

(a) Find \overrightarrow{AB} in the form $x\mathbf{i} + y\mathbf{j}$, where both x and y are given as exact values.

[3 marks]

Question 5b

(b) Find a unit vector in the direction of \overrightarrow{AB} .

[2 marks]

Question 6

In the enchanted kingdom of Vectoria, a magical flying unicorn takes off from the wizard's palace at the point known as O and travels 30 km on a bearing of 300° . Chased by an evil dragon, it then travels an unknown distance of k km due north before reaching the enchanted grove at point P . Given that the position vector of P relative to O is $(x\mathbf{i} + y\mathbf{j})$ km, and that the straight-line distance between the grove and the palace is known to be $30\sqrt{3}$ km, find the exact values of x and y .

[6 marks]

Question 7a

Two forces \mathbf{F}_1 and \mathbf{F}_2 act on a particle, where $\mathbf{F}_1 = 5\mathbf{i} - 3\mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons.

The resultant force \mathbf{R} acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$, and acts in a direction parallel to the vector $(-\mathbf{i} - 3\mathbf{j})$.

- (a) Find the angle between \mathbf{R} and the vector \mathbf{j} , giving your answer in degrees correct to 2 decimal places.

[2 marks]

Question 7b

- (b) Show that $3x - y = -18$.

[3 marks]

Question 7c

- (c) Given that $y = -3$, find the magnitude of \mathbf{R} .

[3 marks]

Question 8

Points A , B and C have position vectors $\vec{OA} = -9\mathbf{i} + 4\mathbf{j}$, $\vec{OB} = -6\mathbf{i}$ and $\vec{OC} = 3\mathbf{i} - 12\mathbf{j}$, respectively.

Use a vector method to show that points A , B and C lie on the same straight line.

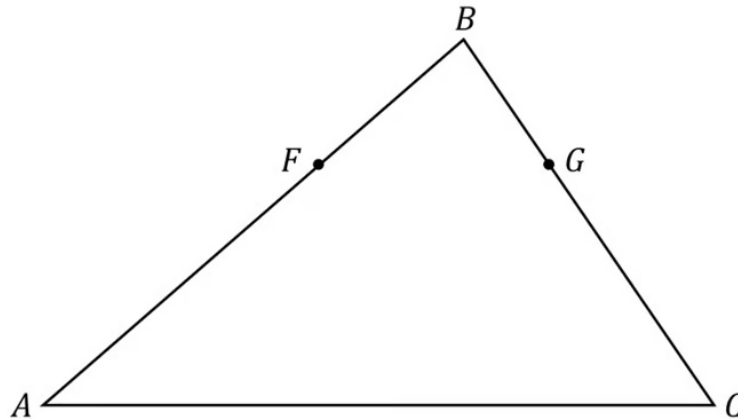
[5 marks]

Question 9a

In triangle ABC , point F lies on AB and point G lies on BC .

F divides AB in the ratio $m:n$.

The line segment FG is parallel to AC .



(a) Explain why $\overrightarrow{BG} = \lambda \overrightarrow{BC}$ for some constant λ , where $0 \leq \lambda \leq 1$.

[1 mark]

Question 9b

(b) Given that $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$, show that

$$\overrightarrow{FG} = \left(\frac{n}{m+n} - \lambda \right) \mathbf{a} + \lambda \mathbf{b}$$

[4 marks]

Question 9c

(c) Using your result from (b), prove that G divides BC in the ratio $n:m$.

[3 marks]